Interphase Mass Transfer for Laminar Concurrent Flow of Carbon Dioxide and Water Between Parallel Plates

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Interphase mass transfer between carbon dioxide gas and water has been investigated. The physical model used was the laminar concurrent flow of gas and liquid in a wide rectangular channel. Because of the high aspect ratio employed, the model was effectively one of parallel plates of infinite width.

Three methods of solving the equations of change—the eigensolution method, the boundary layer theory, and penetration theory—were used to compute the diffusion coefficient of carbon dioxide in water from the experimental data. Calculated values of diffusion coefficients from the penetration theory checked well with those from the eigensolution method, but the values computed from the integrated boundary layer equation were about 10% higher.

Results from the eigensolution agreed within 5% with the data of other investigators in the temperature range of 20° to 30°C.

The current trend in the study of mass transport involves the adaptation of a mathematical model to a real physical situation by means of the general so-called *equations of change*. These complex equations usually must be simplified before they can be solved. The results from such a theoretical analysis can then be compared with experimental data to test the validity of the model. This approach, when properly applied, can lead to a clearer understanding of transport processes.

PREVIOUS WORK

A wide variety of mechanical devices have been employed to study the fundamental factors in interphase mass transfer. One such apparatus which has been used in gas-liquid systems is the wetted-wall column. Mathematical solutions originally derived for laminar heat transfer problems have been applied to mass transfer in the gas stream for the case in which the flow is laminar and the major resistance to mass transfer was in the gas phase. Gilliland and Sherwood (*7*) found that the solution obtained by assuming rodlike flow gave a better fit to the experimental data.

Where the liquid resistance controls, wetted-wall columns are not satisfactory. Pigford (*23*) has derived mathematically the liquid-phase concentration distribution with a laminar parabolic velocity profile; however, the experimental mass transfer coefficients were severalfold of those calculated from his solution. This deviation was due to ripples and surface turbulence. Grimley (*9*) reported that ripples formed at a Reynolds Number as low as 4. Stirba and Hurt (*33*) found that, in addition to the increase of surface due to ripples, turbulence was also present to some degree within the main liquid film and increased the transfer rate. Short wetted-wall columns have been used (*18, 36*) in an attempt to eliminate the formation of ripples, but here the end effects become important.

Davidson and Cullen (*4*) presented a mathematical analysis for a wetted sphere placed in a gas atmosphere. They applied the above-mentioned Pigford solution by using the streamlines as the coordinate framework and found that the calculated diffusion coefficients of carbon dioxide in water checked well with data from other sources.

Theoretical studies on gas bubbles and liquid drops have also been reported (*13, 15*). Again, end effects are important in this type of model; Johnson et al. (*16*) have tried to correlate these effects.

Cullen and Davidson (*2*) among others have studied the mathematical model of a liquid jet entering into a gas. Their novel nozzle design reduced the influence of viscosity so that potential flow could be assumed. Scriven and Pigford (*30*) attacked this problem from the viewpoint of boundary layer theory. Both authors reported close agreement between the experimental and reported absorption rates.

Other gas-liquid contacting devices for fundamental studies that have been reported in the literature involve the rotating drum (*3*), the stirred tank (*8*), the packed column, and the disk column (*32, 37*). In these devices the hydrodynamic characteristics of the liquid-phase are difficult to measure and even more difficult to interpret.

Little work has been reported on gas-liquid mass transfer in horizontal tubes. In one investigation, Westkaemper and White (*38*) reported on the evaporation of practically stagnant carbon tetrachloride into a flowing air stream and correlated their data empirically.

The purpose of this work has been to investigate the absorption of gas into a liquid in a concurrent flow horizontal duct, a simple geometry not previously investigated. It involved two aspects. One was the mathematical analysis of the mass transport characteristics for the specific physical system chosen. The other was the collection of experimental data to verify the theoretical analysis.

A rectangular rather than cylindrical cross section of duct was chosen for simplicity in the mathematical analysis. The gas and the liquid used in this investigation were carbon dioxide and water respectively.

MATHEMATICAL DERIVATIONS

The following assumptions were involved in setting up the mathematical model:

1. Steady state stratified flow existed with a flat interface. Van Rossum (35) has investigated horizontal liquid films at gas-liquid interfaces and found that for a moving film with a thickness of 20 mm. waves developed at air velocities greater than 2 meters/sec. The velocities used in this investigation were considerably lower than the above value.
2. The flow channel was considered to be of infinite width. The aspect ratio used was about 12 to 1.
3. Constant physical properties were assumed. The temperature of the system was essentially constant. Furthermore, since the solubility of carbon dioxide in water is low, the viscosity and density of the liquid phase could be taken as constants. Onda et al. (21) have reported a considerable decrease in the diffusion coefficient of carbon dioxide in water with increasing concentration in the liquid phase. This effect, however, has not been verified as yet.
4. Equilibrium existed at the interface. Emmert and Pigford (5) have reported an apparent interfacial resistance. However, their experiment involved the use of surface active agents. By comparing experimental data with theoretical analysis, both Cullen and Davidson (2) and Harvey and Smith (10) showed that any existing interfacial resistance is negligible when no surfactant is used. The same conclusion was also obtained by Raimondi and Toor (25) and by Scriven and Pigford (29).
5. Molecular diffusion along the direction of flow was assumed to be negligible in comparison with transport owing to bulk flow.

Velocity Distribution in Liquid Phase

For two-phase concurrent stratified flow between parallel plates, the velocity distribution has been given by Russell and Charles (27). For the case of gas-liquid flow in a channel half filled with liquid, the velocity distribution of the liquid layer is

$$\frac{u}{\bar{u}} = 6\left(\frac{\mu_G + \mu_L}{\mu_G + 7\mu_L}\right)\left[\left(\frac{2\mu_L}{\mu_G + \mu_L}\right) - \left(\frac{\mu_L - \mu_G}{\mu_G + \mu_L}\right)\left(\frac{y}{b}\right) - \left(\frac{y}{b}\right)^2\right] \tag{1}$$

The ratio of the volumetric flow rates of the two phases can be obtained as

$$\frac{V_G}{V_L} = \frac{\mu_L(7\mu_G + \mu_L)}{\mu_G(\mu_G + 7\mu_L)} \tag{1a}$$

Concentration Distribution in Liquid Phase

With the assumptions made, the simplified equation of change in the liquid phase is

$$u\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial y^2} \tag{2}$$

and the boundary conditions are

$$c(y, 0) = c_0 \tag{3}$$

$$c(0, z) = c_i \tag{4}$$

$$\frac{\partial c}{\partial y} = 0 \text{ at } y = b \tag{5}$$

It is convenient to make the variables dimensionless by making the following substitutions:

$$w = \frac{u}{\bar{u}}$$

$$\phi = \frac{c_i - c}{c_i - c_0}$$

$$\eta = \frac{y}{b}$$

$$\zeta = \frac{Dz}{\alpha\, b^2 \bar{u}}$$

$$\alpha = \frac{12\mu_L}{\mu_G + 7\mu_L}$$

$$\beta = \frac{\mu_L - \mu_G}{2\mu_L}$$

$$\gamma = \frac{\mu_L + \mu_G}{2\mu_L}$$

Equations (1) to (5) become

$$w = \alpha(1 - \beta\eta - \gamma\eta^2) \tag{6}$$

$$\frac{\partial \phi}{\partial \zeta} = \frac{1}{(1 - \beta\eta - \gamma\eta^2)}\frac{\partial^2 \phi}{\partial \eta^2} \tag{7}$$

$$\phi(\eta, 0) = 1 \tag{8}$$

$$\phi(0, \zeta) = 0 \tag{9}$$

$$\frac{\partial \phi}{\partial \eta} = 0 \text{ at } \eta = 1 \tag{10}$$

Three methods—the eigensolution, the boundary layer method, and the penetration theory—were used to solve this problem.

EIGENSOLUTIONS

The general solution of Equations (7) to (10) can be written as

$$\phi = \sum_{n=1}^{\infty} A_n \exp(-\lambda_n^2 \zeta)\, R_n(\eta) \tag{11}$$

where λ_n and R_n satisfy the Stürm-Liouville system:

$$\frac{d^2R_n}{d\eta^2} + \lambda_n^2(1 - \beta\eta - \gamma\eta^2)R_n = 0 \tag{12}$$

$$R_n(0) = 0 \tag{13}$$

$$\frac{dR_n}{d\eta} = 0 \text{ at } \eta = 1 \tag{14}$$

Equation (12) is a second-order differential equation; hence, there are two independent solutions. The second solution, however, is undefined at $\eta = 0$ and thus is not important here.

The eigenvalues λ_n and the eigenfunctions R_n could be obtained by the method of Frobenius to expand R_n in series form. However, with the aid of a high-speed computer, it is much easier to solve for them numerically. The coefficients A_n can be evaluated by use of the initial condition at $\zeta = 0$:

$$1 = \sum_{n=1}^{\infty} A_n R_n$$

which gives, since the functions R_n form a complete orthogonal set in the interval (0, 1)

$$A_n = \frac{\int_0^1 (1 - \beta\eta - \gamma\eta^2)R_n d\eta}{\int_0^1 (1 - \beta\eta - \gamma\eta^2)R_n^2 d\eta} \tag{15}$$

The cup-mixing concentration can be defined as the concentration which any quantity of the liquid passing the cross-sectional area of the channel in a time interval would assume if it were collected and perfectly mixed in a cup. At the end of the duct, this value $\overline{c}_L$ is given by

$$\overline{c}_L = c_i - (c_i - c_0) \sum_{n=1}^{\infty} A_n \exp(-\lambda_n^2 \zeta_L) \overline{R}_n \qquad (16)$$

where ζ_L is evaluated at $z = L$, the length of the duct, and

$$\overline{R}_n = \frac{\int_0^1 w R_n d\eta}{\int_0^1 w d\eta} \qquad (17)$$

$$\overline{R}_n = \int_0^1 \alpha(1 - \beta\eta - \gamma\eta^2) R_n d\eta \qquad (18)$$

With numerical solutions, both A_n and $\overline{R}_n$ can be obtained by numerical integration.

It should be mentioned here that a fully developed velocity profile was used in the above derivations. If uniform velocity is assumed in each phase at the inlet, the length of the entrance region can be estimated by using Blasius's solution for the boundary layer along a flat plate (*1*) at the wall and Potter's solution for the boundary layer at an interface (*24*). The estimated value of the distance required to reach a steady state velocity profile for the liquid phase was less than 4 in., which was small in comparison with the total length used.

BOUNDARY-LAYER SOLUTION

The concentration distribution at the end of the duct, calculated by use of eigensolutions, showed that absorption occurred in a distance less than 25% of the liquid depth from the gas-liquid interface. This indicated that the mass transfer process for the system was essentially a boundary layer problem. An approximate solution, the von Kármán integrated method (*17*), can be developed as follows.

A simple material balance for a small increment in z, Δz, in the boundary layer gives

$$\frac{d}{dz} \int_0^\delta u c \, dy = -D \left(\frac{\partial c}{\partial y}\right)_{y=0} \qquad (19)$$

Using the following substitution for c in Equation (19)

$$c = (1 - \phi)(c_i - c_0) + c_0$$

yields

$$(c_i - c_0) \frac{d}{dz} \int_0^\delta (1 - \phi) u \, dy + c_0 \frac{d}{dz} \int_0^\delta u \, dy = (c_i - c_0) D \left(\frac{\partial \phi}{\partial y}\right)_{y=0} \qquad (20)$$

For a fully developed velocity profile, u is independent of z; hence, the second term becomes zero. This gives

$$\frac{d}{dz} \int_0^\delta (1 - \phi) u \, dy = D \left(\frac{\partial \phi}{\partial y}\right)_{y=0} \qquad (21)$$

If the concentration function ϕ is approximated by a third-degree polynomial in y/δ, if at $y = 0$ $\phi = 0$ and $\partial^2\phi/\partial y^2 = 0$ (the latter is a smooth-fit condition), and if at $y = \delta$ $\phi = 1$ and $\partial\phi/\partial y = 0$, then

$$\phi = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad (22)$$

and

$$\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = \frac{3}{2\delta} \qquad (23)$$

Then Equation (21) can be integrated by use of the velocity-distribution Equation (6) and Equations (22) and (23). Since $\delta = 0$ at $z = 0$ the result can be further integrated with respect to z:

$$90\left(\frac{\delta}{b}\right)^2 - 16\beta\left(\frac{\delta}{b}\right)^3 - 5\gamma\left(\frac{\delta}{b}\right)^4 = 720 \frac{Dz}{\alpha b^2 \overline{u}} \qquad (24)$$

When δ/b is small, the second and third terms in Equation (24) may be neglected, since the values of β and γ for the case of carbon dioxide absorption are about 0.5, and the boundary layer thickness may be approximated by

$$\delta = 2.828 \sqrt{Dz/\alpha\overline{u}} \qquad (25)$$

The local mass flux, denoted by q_z, given by

$$q_z = -D \left(\frac{\partial c}{\partial y}\right)_{y=0} = D(c_i - c_0) \left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

when combined with Equations (23) and (25), leads to

$$q_z = 0.530 (c_i - c_0) \sqrt{\alpha \overline{u} D/z} \qquad (26)$$

Total mass transfer per unit time, Q, may be obtained by integrating Equation (26) from $z = 0$ to $z = L$. With the width of the duct equal to a,

$$Q = 1.061\, a (c_i - c_0) \sqrt{\alpha \overline{u}\, DL} \qquad (27)$$

The volumetric flow rate is given by

$$V = \overline{u}\, a\, b \qquad (28)$$

From Equations (27) and (28) the final concentration is obtained as

$$\overline{c}_L = 1.061\, (c_i - c_0) \sqrt{\alpha DL/b^2 \overline{u}} \qquad (29)$$

PENETRATION THEORY

The neglect of the second and third terms in Equation (24) is equivalent to the assumption of constant velocity in the boundary layer. With this assumption, Higbie's penetration theory for gas absorption (*11*) can be applied, which gives

$$q_z = (c_i - c_0) \sqrt{\alpha \overline{u} D/\pi z}$$

The final concentration is then given by

$$\overline{c}_L = 1.136\, (c_i - c_0) \sqrt{\alpha DL/b^2 \overline{u}} \qquad (30)$$

A comparison between Equation (29) and Equation (30) shows that Higbie's penetration theory gives a final concentration about 7% higher than that obtained from the boundary layer solution.

EXPERIMENTAL APPARATUS

The experimental apparatus consisted of an absorption channel, a gas and liquid feed system, and the liquid sampling device. A schematic diagram is shown in Figure 1.

The inside dimensions of the channel were ⅞ in. high by 5⅜ in. wide. Total contact length was 6 ft. No waves at the gas-liquid interface were observed. Since the liquid drawoff section was tapered and since the total contacting

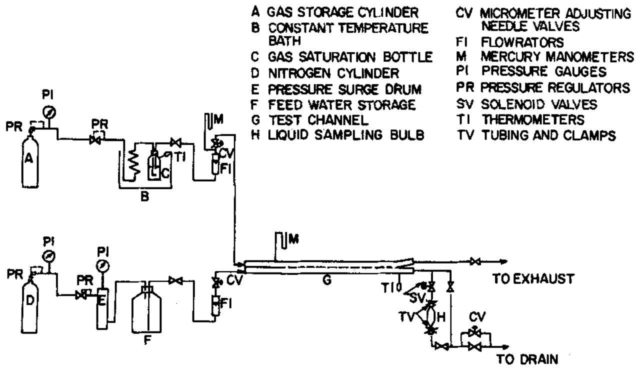

Fig. 1. Schematic flow diagram of experimental apparatus.

length was large, the effect of the liquid drawoff on the upstream liquid-phase velocity distribution in the test section was small.

Liquid and gas flows were thermostated and measured by flowrators accurate to within 2% of the maximum flow. Control of the liquid drawoff rate was essential to keep the liquid level in the test channel at exactly one half of its height. The liquid feed rate was set, and then the gas feed rate was controlled to a predetermined ratio based on the quantity $\mu_L\ (7\mu_G + \mu_L)/\ \mu_G\ (\mu_G + 7\mu_L)$.

When the amount of dissolved carbon dioxide in water was analyzed, the standard method of reacting with an excess amount of barium hydroxide and back titrating with hydrochloric acid was employed.

CALCULATIONS AND RESULTS

Numerical Solution of the Eigenvalue Problem

The Stürm-Liouville system, Equations (12) to (14), was solved numerically on a digital computer by the Runge-Kutta method. In order to compute the concentration distribution at the end of the channel, a rather fine mesh size, $\Delta\eta = 0.02$, was used. The numerical values of the eigenfunctions R_n were integrated by Simpson's rule to compute A_n and $\overline{R}_n$. Ten eigenvalues with the associated eigenfunctions were found to be enough for the computation.

Theoretically, the eigenvalues and the eigenfunctions are dependent upon the constants β and γ, which are functions of the viscosities of the liquid and the gas. However, since the viscosity of the liquid was much greater than that of the gas and the temperature range used in this investigation was small, these physical parameters were fairly constant. The values at 25°C. were therefore used in all the computations.

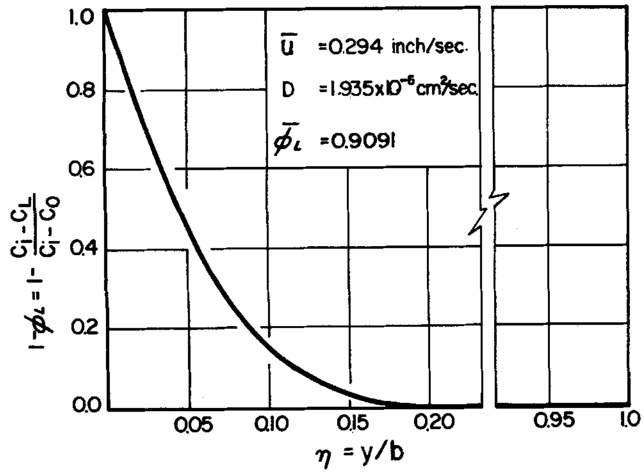

Fig. 2. Concentration profile at the end of the test section.

TABLE 1. VALUES OF VISCOSITY PARAMETERS

Temperature, °C.	α	β	γ
20	1.7107	0.4927	0.5073
25	1.7102	0.4917	0.5083
30	1.7097	0.4906	0.5094

TABLE 2. VALUES OF λ_n, A_n, $\overline{R}_n$ AT 25°C.

λ_n	A_n	$\overline{R}_n$
2.4854	2.8123	0.27685
6.7970	2.7511	0.03702
11.102	2.7441	0.01387
15.407	2.7425	0.00720
19.712	2.7433	0.00440
24.019	2.7476	0.00297
28.328	2.7579	0.00213
32.642	2.7796	0.00161
36.962	2.8196	0.00126
41.292	2.8905	0.00101

Table 1 shows the values of the viscosity parameters α, β, and γ for three different temperatures. The first ten eigenvalues, together with the associated values of A_n and $\overline{R}_n$, are given in Table 2 .

Calculation of the Diffusion Coefficient

The methods using the boundary-layer solution and the penetration theory were straightforward applications of Equations (29) and (30) respectively. Pure distilled water was used as the feed liquid; hence $c_0 = 0$. The equilibrium interface concentrations, c_i, were obtained from the equilibrium data of Morrison and Billett (*19*) and corrected to the carbon dioxide partial pressure by assuming Henry's Law held. The eigensolution method involved trial and error. The experimental data and the calculated diffusion coefficients are tabulated in Appendix A.*

Calculation of Concentration Distribution

The fine mesh size used enabled the calculation of a concentration distribution from Equation (11). The computed values of the concentration function ϕ at the end of the channel for one experimental run are plotted in Figure 2, which illustrates that only about 20% of the liquid depth had been penetrated by the gas at the end of the channel.

DISCUSSION OF RESULTS

Accuracy of the Data and Results

Four sources of error were involved in the gathering of the experimental data and in the calculation of the diffusion coefficients. These are discussed separately as follows.

Errors in the Mathematical Model The mathematical derivations were based on the assumption of flow between parallel plates of infinite width. This can never be realized in actual practice; however, the higher the aspect ratio used, the more closely the experiment fits the assumption.

To estimate the effect due to the presence of the side walls, the two-dimensional velocity distribution for fully developed laminar flow was solved numerically by a finite-difference method, a variational principle being used to obtain the interface conditions (6). The entire width

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* Appendix A to this paper has been deposited as document 7635 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or for 35-mm microfilm.

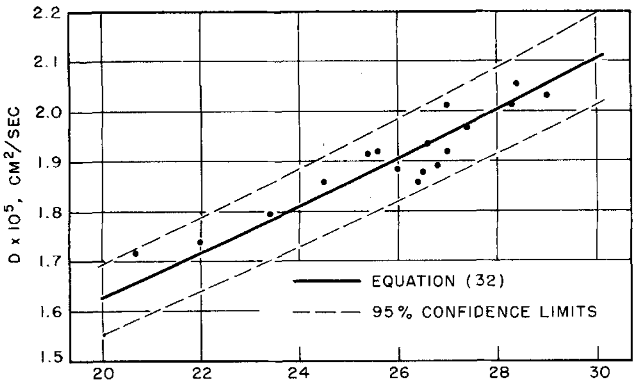

Fig. 3. Correlation of experimental data.

of the channel was considered as being composed of forty sections of equal widths. Volumetric flow rates and the interfacial velocities were computed for each section, and the penetration theory was applied to each section to calculate the final concentration for that section. The cup-mixing final concentration for the entire channel was then obtained, and the result compared with that obtained when the penetration theory was applied to the entire channel, with the infinite parallel-plate model still assumed. The final concentration computed by the above method was 1.05% lower than that obtained for the assumed parallel-plate model. This meant that the calculated values of the diffusion coefficient were low and should be multiplied by a factor of 1.02.

Errors due to Experimental Apparatus In the study of mass transfer in a flow system, end effects are always present. These effects are reduced when the length of the contacting equipment is increased. The actual contact length used in this investigation was 72 in., which, as discussed above, was much greater than the estimated length of the entrance region. Therefore, the end effects in this apparatus should have been small.

The liquid flowrator used had been calibrated to be accurate within 1%; the accuracy of the gas flowrator was claimed to be within 2% of the maximum flow. For runs at lower velocity, errors in flow measurement could be significant. A higher gas velocity than measured would result in a higher interface velocity, thus decreasing the amount of gas absorbed.

The failure to control the gas-liquid interface at exactly half of the channel height was another possible source of error. Any shift of the location of the interface would have affected the velocity of flow, which in turn would have affected the amount of mass transfer.

From the discussion in Appendix B* it has been concluded that natural convective mixing caused by density differences was not a problem.

Errors in the Analysis of Liquid Samples It is rather difficult to analyze accurately the small amount of carbon dioxide contained in water. Since the diffusion coefficient is approximately proportional to the square of the final concentration, an error of 1% in the determination of the final concentration would result in a 2% error in the calculated diffusion coefficient.

Errors in the Equilibrium Data. The equilibrium solubility data used in the calculation of the diffusion coefficients were taken from Seidell (*31*), who has collected data of various investigators. Most of them check quite well; some, however, show a possible deviation of ±1.5%.

Comparison of the Three Calculation Methods

If enough terms are used, the eigensolution gives an accurate approximation of the solution of the boundary-value problem. Therefore, if the assumptions made were valid, this method should give results more accurate than those of the other methods.

The boundary layer solution, being based on an assumed concentration distribution which might not be true for the actual case, should provide less accurate results. However, in a large number of fluid flow and heat transfer problems, the boundary layer solutions obtained by integrated methods are usually accurate within ±10%.

It is interesting to note that the results based on the penetration theory checked well with those obtained from the eigensolution. The velocity distribution in the liquid phase computed by a finite-difference method revealed that the change of velocity near the gas-liquid interface was small. In these circumstances, use of the penetration theory is justified.

Comparison of Results with Those of Other Investigators

For the reason discussed above, the results obtained by the eigensolution method will be used to compare values of the diffusion coefficient with those obtained by other investigators. There is considerable divergence among the reported diffusion coefficients and so the correct value is not known. Therefore, rather than selecting certain values of the diffusion coefficients from the literature and using these to calculate theoretical values of the outlet concentrations, which can be then compared to the experimental values, a more appropriate basis for comparison is to use the experimental outlet concentrations to calculate the diffusion coefficients and compare these latter with other investigators' values.

The calculated diffusion coefficients in this work have been correlated vs. temperature, with the Nernst-Einstein relationship. This gave

$$D\mu_L/T = 5.56 \times 10^{-8} \qquad (31)$$

Equation (31) is plotted in Figure 3 together with curves showing the 95% confidence limits and the calculated values of the diffusion coefficients. The scattering of the data points indicates that the reproducibility of the present method is no better than 5%. This is not unsatisfactory in view of the divergence among the reported diffusion coefficients in the literature. For example, with a sintered cell method, Peaceman (*22*) reported that the measured values of the diffusion coefficient at 30°C. ranged from 2.19×10^{-5} to 2.42×10^{-5}, and the values measured by Scriven (*28*) at 25°C. ranged from 1.80×10^{-5} to 1.94×10^{-5}.

Figure 4 compares Equation (31) with the diffusion coefficients reported by other investigators. A second

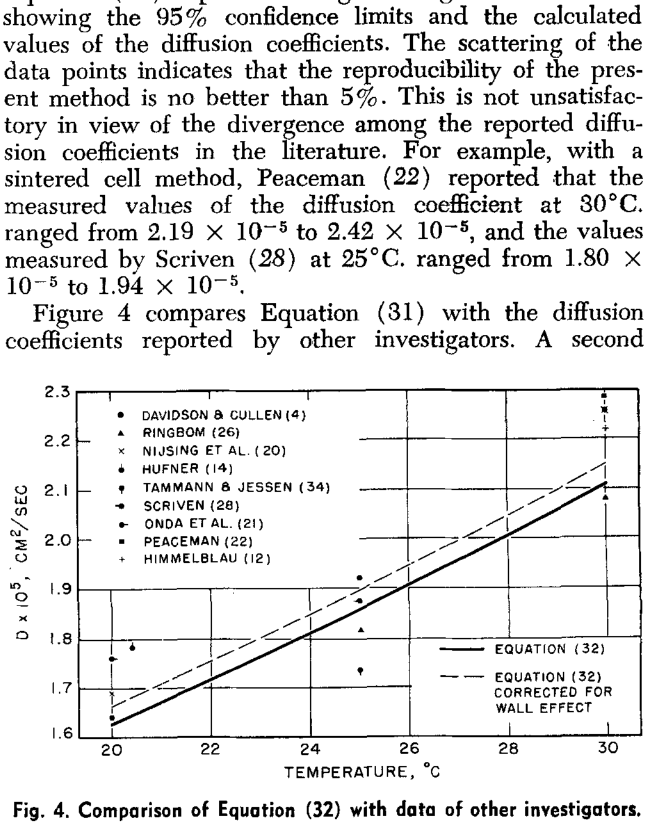

Fig. 4. Comparison of Equation (32) with data of other investigators.

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* Appendix B to this paper has been deposited as document 7635 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or for 35-mm. microfilm.

curve, higher than the curve for Equation (31), is the correlation of the diffusion coefficients after the correction for the wall effect has been made. Generally, the results of this work check with the data of other investigators within 5%.

CONCLUSION

The theoretical study of mass transfer in a gas-liquid system by the method of laminar concurrent flow in a horizontal duct seems to be quite satisfactory. The experimental measurements agreed well with theoretical predictions. Since the viscosities of gases are generally small in comparison with those of liquids, it is expected that the eigensolution obtained in this work can be applied to gas-water systems other than carbon dioxide.

ACKNOWLEDGMENT

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NOTATION

A = constant
a = width of the test channel, cm.
b = one-half of the height of the test channel, cm.
c = concentration of carbon dioxide in water, mg./cc.
$\bar{c}$ = cup-mixing concentration of carbon dioxide in water, mg./cc.
C_1 = constant
C_2 = constant
D = diffusion coefficient of carbon dioxide in water, sq.cm./sec.
L = length of the test section, cm.
Q = total mass transfer per unit time, mg./sec.
q = local mass flux, mg./(sq.cm.) (sec.)
R = eigenfunction
$\bar{R}$ = cup-mixing value of R
T = absolute temperature, °K.
u = velocity of the liquid in the z direction (direction of flow), cm./sec.
$\bar{u}$ = average velocity of the liquid, cm./sec.
V = volumetric flow rate of the liquid, cc./sec.
w = dimensionless velocity term, $u/\bar{u}$
y = distance from the gas-liquid interface, positive downward, cm.
z = distance from the entrance of the test section, cm.

Greek Letters

α = viscosity parameter, $12\mu_L/(\mu_G + 7\mu_L)$
β = viscosity parameter, $(\mu_L - \mu_G)/2\mu_L$
γ = viscosity parameter, $(\mu_L + \mu_G)/2\mu_L$
δ = boundary layer thickness, cm.
ζ = dimensionless variable, $Dz/ab^2\bar{u}$
η = dimensionless variable, y/b
λ = eigenvalue
μ = absolute viscosity, centipoises
π = constant, = 3.14159
ϕ = concentration function, $(c_i - c)/(c_i - c_0)$
$\bar{\phi}$ = cup-mixing value of ϕ

Subscripts

i = gas-liquid interface, or equilibrium
G = gas
L = liquid (in μ_L), or end of the test section (in c_L, ϕ_L)
n = positive integers
0 = entrance of the test section
z = at location z

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